6 If A$(-1, 1, 2)$, B$(2, 0, 1)$ and C$(k, 2, -1)$ are three points in space, find k if the area of triangle ABC is $\sqrt{88}$ units^2.

7 A, B and C are three points with position vectors **a**, **b** and **c** respectively. Find a formula for S, the total surface area of the tetrahedron OABC.

8 Three distinct points, A, B and C, have position vectors **a**, **b** and **c** respectively. Prove that A, B and C are collinear $\Leftrightarrow$ $(\mathbf{b}-\mathbf{a}) \times (\mathbf{c}-\mathbf{b}) = \mathbf{0}$.

TEST FOR COPLANAR POINTS

Four points in space are either coplanar or form the vertices of a tetrahedron. If they are coplanar, the volume of the tetrahedron is zero. So,

> if four points A, B, C and D have position vectors **a**, **b**, **c** and **d** respectively then A, B, C and D are coplanar $\Leftrightarrow$ $(\mathbf{b}-\mathbf{a}) \bullet (\mathbf{c}-\mathbf{a}) \times (\mathbf{d}-\mathbf{a}) = 0$.

Example 46

Are the points A$(1, 2, -4)$, B$(3, 2, 0)$, C$(2, 5, 1)$ and D$(5, -3, -1)$ coplanar?

$$\mathbf{b}-\mathbf{a} = \overrightarrow{AB} = \begin{bmatrix} 3-1 \\ 2-2 \\ 0--4 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \\ 4 \end{bmatrix} \qquad \mathbf{c}-\mathbf{a} = \overrightarrow{AC} = \begin{bmatrix} 2-1 \\ 5-2 \\ 1--4 \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \\ 5 \end{bmatrix}$$

$$\mathbf{d}-\mathbf{a} = \overrightarrow{AD} = \begin{bmatrix} 5-1 \\ -3-2 \\ -1--4 \end{bmatrix} = \begin{bmatrix} 4 \\ -5 \\ 3 \end{bmatrix}$$

and $(\mathbf{b}-\mathbf{a}) \bullet (\mathbf{c}-\mathbf{a}) \times (\mathbf{d}-\mathbf{a}) = \begin{vmatrix} 2 & 0 & 4 \\ 1 & 3 & 5 \\ 4 & -5 & 3 \end{vmatrix}$

$= 2(9+25) + 4(-5-12)$

$= 0$

$\therefore$ A, B, C and D are coplanar.

9 Are these points coplanar?

a A$(1, 1, 2)$, B$(2, 4, 0)$, C$(3, 1, 1)$ and D$(4, 0, 1)$

b P$(2, 0, 5)$, Q$(0, -1, 4)$, R$(2, 1, 0)$, S$(1, 1, 1)$

10 Find k given that A$(2, 1, 3)$, B$(4, 0, 1)$, C$(0, k, 2)$, D$(1, 2, -1)$ are coplanar.

REVIEW SET 15A (MAINLY 2-D)

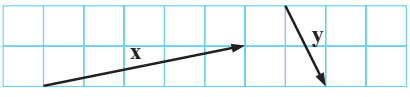
1 Using a scale of 1 cm represents 10 units, sketch a vector to represent:

a an aeroplane taking off at an angle of 8^o to the runway with a speed of 60 m/s

b a displacement of 45 m in a direction 060^o.

2 Copy the given vectors and find geometrically:

a $\mathbf{x}+\mathbf{y}$ **b** $\mathbf{y}-2\mathbf{x}$



3 Find a single vector which is equal to: **a** $\overrightarrow{PR} + \overrightarrow{RQ}$ **b** $\overrightarrow{PS} + \overrightarrow{SQ} + \overrightarrow{QR}$

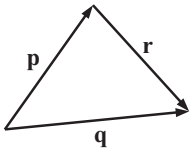
4 Dino walks for 9 km in a direction 246^o and then for 6 km in a direction 096^o. Find his displacement from his starting point.

5 Simplify **a** $\overrightarrow{AB} - \overrightarrow{CB}$ **b** $\overrightarrow{AB} + \overrightarrow{BC} - \overrightarrow{DC}$.

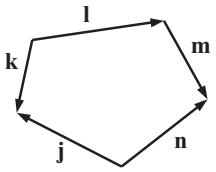
6 What geometrical facts can be deduced from the equations:

a $\overrightarrow{AB} = \frac{1}{2}\overrightarrow{CD}$ **b** $\overrightarrow{AB} = 2\overrightarrow{AC}$?

7 Construct vector equations for: **a**



b



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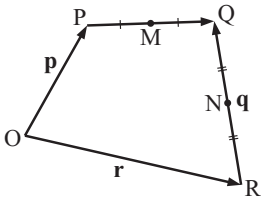


In the figure alongside $\overrightarrow{OP} = \mathbf{p}$, $\overrightarrow{OR} = \mathbf{r}$ and $\overrightarrow{RQ} = \mathbf{q}$.

If M and N are midpoints of the sides as shown, find in terms of **p**, **q** and **r**:

a $\overrightarrow{OQ}$ **b** $\overrightarrow{PQ}$ **c** $\overrightarrow{ON}$ **d** $\overrightarrow{MN}$

9 Draw arrow diagrams to represent: **a** $\begin{bmatrix} 4 \\ 3 \end{bmatrix}$ **b** $\begin{bmatrix} 3 \\ -5 \end{bmatrix}$ **c** $\begin{bmatrix} 0 \\ -4 \end{bmatrix}$

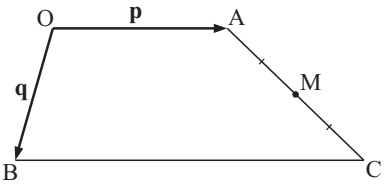
10 If $\mathbf{p} = \begin{bmatrix} -3 \\ 1 \end{bmatrix}$, $\mathbf{q} = \begin{bmatrix} 2 \\ -4 \end{bmatrix}$, and $\mathbf{r} = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$ find:

a $2\mathbf{p} + \mathbf{q}$ **b** $\mathbf{q} - 3\mathbf{r}$ **c** $\mathbf{p} - \mathbf{q} + \mathbf{r}$

11 If $\overrightarrow{PQ} = \begin{bmatrix} -4 \\ 1 \end{bmatrix}$, $\overrightarrow{RQ} = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$ and $\overrightarrow{RS} = \begin{bmatrix} 2 \\ -3 \end{bmatrix}$, find $\overrightarrow{SP}$.

12 If $\mathbf{r} = \begin{bmatrix} 4 \\ 1 \end{bmatrix}$ and $\mathbf{s} = \begin{bmatrix} -3 \\ 2 \end{bmatrix}$ find: **a** $|\mathbf{r}|$ **b** $|\mathbf{s}|$ **c** $|\mathbf{r} + \mathbf{s}|$ **d** $|2\mathbf{s} - \mathbf{r}|$

13



BC is parallel to OA and is twice its length. Find in terms of **p** and **q** vector expressions for **a** $\overrightarrow{AC}$ **b** $\overrightarrow{OM}$.

14 If $\mathbf{p} = \begin{bmatrix} -3 \\ 1 \end{bmatrix}$, $\mathbf{q} = \begin{bmatrix} 2 \\ -4 \end{bmatrix}$ and $\mathbf{r} = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$, find $\mathbf{x}$ if: **a** $\mathbf{p} - 3\mathbf{x} = \mathbf{0}$ **b** $2\mathbf{q} - \mathbf{x} = \mathbf{r}$

15 Use vectors to show that WYZX is a parallelogram if X is $(-2, 5)$, Y$(3, 4)$, W$(-3, -1)$, and Z$(4, 10)$.

16 Find scalars r and s such that $r\begin{bmatrix} -2 \\ 1 \end{bmatrix} + s\begin{bmatrix} 3 \\ -4 \end{bmatrix} = \begin{bmatrix} 13 \\ -24 \end{bmatrix}$.

17 AB and CD are diameters of a circle centre O. If $\overrightarrow{OC} = \mathbf{q}$ and $\overrightarrow{OB} = \mathbf{r}$, find:

a $\overrightarrow{DB}$ in terms of **q** and **r** **b** $\overrightarrow{AC}$ in terms of **q** and **r**.

What can be deduced about DB and AC?

REVIEW SET 15B (MAINLY 3-D)

1 Given P(2, −5, 6) and Q(−1, 7, 9), find:

a the position vector of Q from P **b** the distance from P to Q

c the distance from P to the x-axis.

2 For $\mathbf{m} = \begin{bmatrix} 6 \\ -3 \\ 1 \end{bmatrix}$, $\mathbf{n} = \begin{bmatrix} 2 \\ 3 \\ -4 \end{bmatrix}$ and $\mathbf{p} = \begin{bmatrix} -1 \\ 3 \\ 6 \end{bmatrix}$, find:

a $\mathbf{m} - \mathbf{n} + \mathbf{p}$ **b** $2\mathbf{n} - 3\mathbf{p}$ **c** $|\mathbf{m} + \mathbf{p}|$

3 If $\overrightarrow{AB} = \begin{bmatrix} 2 \\ -7 \\ 4 \end{bmatrix}$ and $\overrightarrow{AC} = \begin{bmatrix} -6 \\ 1 \\ -3 \end{bmatrix}$, find $\overrightarrow{CB}$.

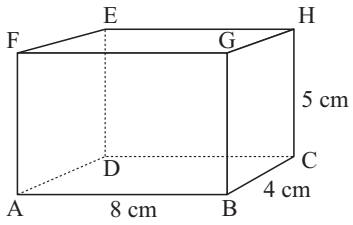
4 Find m and n if $\begin{bmatrix} 3 \\ m \\ n \end{bmatrix}$ and $\begin{bmatrix} -12 \\ -20 \\ 2 \end{bmatrix}$ are parallel vectors.

5 Prove that P(−6, 8, 2), Q(4, 6, 8) and R(19, 3, 17) are collinear. Hence find the ratio in which Q divides PR.

6 Find t if $\begin{bmatrix} -4 \\ t+2 \\ t \end{bmatrix}$ and $\begin{bmatrix} t \\ 1+t \\ -3 \end{bmatrix}$ are perpendicular vectors.

7 Determine the angle between $\begin{bmatrix} 2 \\ -4 \\ 3 \end{bmatrix}$ and $\begin{bmatrix} -1 \\ 1 \\ 3 \end{bmatrix}$.

8 Find the measure of angle GAC in the rectangular box alongside. Use vector methods.



9 For P(2, 3, −1) and Q(−4, 4, 2) find:

a $\overrightarrow{PQ}$ **b** the distance between P and Q **c** the midpoint of PQ.

10 For $\mathbf{p} = \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix}$, $\mathbf{q} = \begin{bmatrix} 3 \\ -1 \\ 4 \end{bmatrix}$ and $\mathbf{r} = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$ find:

a $\mathbf{p} \bullet \mathbf{q}$ **b** $\mathbf{p} + 2\mathbf{q} - \mathbf{r}$ **c** the angle between $\mathbf{p}$ and $\mathbf{r}$.

11 Find all angles of the triangle with vertices K(3, 1, 4), L(−2, 1, 3) and M(4, 1, 3).

12 Find the angle between $\begin{bmatrix} 3 \\ 1 \\ -2 \end{bmatrix}$ and $\begin{bmatrix} 2 \\ 5 \\ 1 \end{bmatrix}$.

13 If A(4, 2, −1), B(−1, 5, 2), C(3, −3, c) and triangle ABC is right angled at B, find possible values of c.

14 Explain why:

a $\mathbf{a} \bullet \mathbf{b} \bullet \mathbf{c}$ is meaningless **b** you do not need a bracket for $\mathbf{a} \bullet \mathbf{b} \times \mathbf{c}$.

15 Find k if the following are unit vectors: **a** $\begin{bmatrix} \frac{4}{7} \\ \frac{1}{k} \end{bmatrix}$ **b** $\begin{bmatrix} k \\ k \end{bmatrix}$

REVIEW SET 15C (MAINLY 2-D)

1 If $\mathbf{p} = \begin{bmatrix} 3 \\ -2 \end{bmatrix}$, $\mathbf{q} = \begin{bmatrix} -1 \\ 5 \end{bmatrix}$, and $\mathbf{r} = \begin{bmatrix} -3 \\ 4 \end{bmatrix}$ find: **a** $\mathbf{p} \bullet \mathbf{q}$ **b** $\mathbf{q} \bullet (\mathbf{p} - \mathbf{r})$

2 Using $\mathbf{p} = \begin{bmatrix} 3 \\ -2 \end{bmatrix}$, $\mathbf{q} = \begin{bmatrix} -2 \\ 5 \end{bmatrix}$ and $\mathbf{r} = \begin{bmatrix} 1 \\ -3 \end{bmatrix}$ verify that:
$\mathbf{p} \bullet (\mathbf{q} - \mathbf{r}) = \mathbf{p} \bullet \mathbf{q} - \mathbf{p} \bullet \mathbf{r}$.

3 Determine the value of t if $\begin{bmatrix} 3 \\ 3 - 2t \end{bmatrix}$ and $\begin{bmatrix} t^2 + t \\ -2 \end{bmatrix}$ are perpendicular.

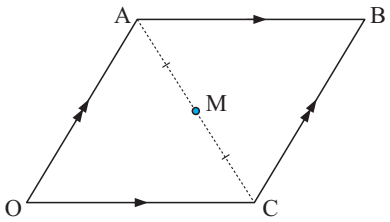
4 Given A(2, 3), B(−1, 4) and C(3, k), find k if $\measuredangle$BAC is a right angle.

5 Find all vectors which are perpendicular to the vector $\begin{bmatrix} -4 \\ 5 \end{bmatrix}$.

6 Find the measure of all angles of triangle KLM for K(−2, 1), L(3, 2) and M(1, −3).

7 Find the angle between the two lines with equations $4x - 5y = 11$ and $2x + 3y = 7$.

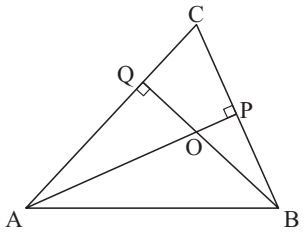
8 **a** **Do not** assume any diagonal properties of parallelograms. OABC is a parallelogram with $\overrightarrow{OA} = \mathbf{p}$ and $\overrightarrow{OC} = \mathbf{q}$. M is the midpoint of AC.



i Find in terms of $\mathbf{p}$ and $\mathbf{q}$:
(1) $\overrightarrow{OB}$ **(2)** $\overrightarrow{OM}$

ii Show using **i** only that O, M and B are collinear and M is the midpoint of OB.

b AP and BQ are altitudes of triangle ABC.
Let $\overrightarrow{OA} = \mathbf{p}$, $\overrightarrow{OB} = \mathbf{q}$ and $\overrightarrow{OC} = \mathbf{r}$.



i Find vector expressions for $\overrightarrow{AC}$ and $\overrightarrow{BC}$ in terms of $\mathbf{p}$, $\mathbf{q}$ and $\mathbf{r}$.

ii Deduce that $\mathbf{q} \bullet \mathbf{r} = \mathbf{p} \bullet \mathbf{q} = \mathbf{p} \bullet \mathbf{r}$.

iii Hence prove that OC is perpendicular to AB.

9 If $\mathbf{a} = \begin{bmatrix} 2 \\ -3 \\ 1 \end{bmatrix}$, $\mathbf{b} = \begin{bmatrix} -1 \\ 2 \\ 3 \end{bmatrix}$, find:

a $2\mathbf{a} - 3\mathbf{b}$ **b** $\mathbf{x}$ if $\mathbf{a} - 3\mathbf{x} = \mathbf{b}$ **c** the projection vector of $\mathbf{a}$ on $\mathbf{b}$.

10 If $|\mathbf{a}| = 3$, $|\mathbf{b}| = \sqrt{7}$ and $\mathbf{a} \times \mathbf{b} = \mathbf{i} + 2\mathbf{j} - 3\mathbf{k}$ find:

a $\mathbf{a} \bullet \mathbf{b}$ **b** the area of triangle OAB given that $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$

c the volume of tetrahedron OABC if C is the point (1, −1, 2).

REVIEW SET 15D (MAINLY 3-D)

Click on the icon to obtain printable review sets and answers

c A, B and C are collinear and AB = 2 BC
d A, B and C are collinear and AC = 3 BC

5 **a** $\overrightarrow{PR} = \begin{bmatrix}-1\\-3\\3\end{bmatrix}$, $\overrightarrow{QS} = \begin{bmatrix}-2\\-6\\6\end{bmatrix}$ **b** PR = $\frac{1}{2}$ QS

EXERCISE 15I

1 **a** $\begin{bmatrix}1\\-1\\1\end{bmatrix}$, $\sqrt{3}$ units **b** $\begin{bmatrix}3\\-1\\1\end{bmatrix}$, $\sqrt{11}$ units

c $\begin{bmatrix}1\\0\\-5\end{bmatrix}$, $\sqrt{26}$ units **d** $\begin{bmatrix}0\\\frac{1}{2}\\\frac{1}{2}\end{bmatrix}$, $\frac{1}{\sqrt{2}}$ units

2 **a** $k = \pm 1$ **b** $k = \pm 1$ **c** $k = 0$ **d** $k = \pm\frac{\sqrt{11}}{4}$ **e** $k = \pm\frac{2}{3}$

3 **a** 5 units **b** $\sqrt{6}$ units **c** 3 units **d** $\div$ 6.12 units

4 **a** $\frac{1}{\sqrt{5}}(\mathbf{i} + 2\mathbf{j})$ **b** $\frac{1}{\sqrt{13}}(2\mathbf{i} - 3\mathbf{k})$ **c** $\frac{1}{\sqrt{33}}(-2\mathbf{i} - 5\mathbf{j} - 2\mathbf{k})$

5 **a** $\frac{3}{\sqrt{5}}\begin{bmatrix}2\\-1\end{bmatrix}$ **b** $-\frac{2}{\sqrt{17}}\begin{bmatrix}-1\\-4\end{bmatrix}$ **c** $\frac{6}{\sqrt{18}}\begin{bmatrix}-1\\4\\1\end{bmatrix}$ **d** $-\frac{5}{3}\begin{bmatrix}-1\\-2\\-2\end{bmatrix}$

EXERCISE 15J.1

1 **a** 7 **b** 22 **c** 29 **d** 66 **e** 52 **f** 3 **g** 5 **h** 1

2 **a** 2 **b** 2 **c** 14 **d** 14 **e** 4 **f** 4

3 **a** 1 **b** 1 **c** 0

5 **a** $t = 6$ **b** $t = -8$ **c** $t = 0$ or 2 **d** $t = -\frac{3}{2}$

6 **a** $t = -\frac{3}{2}$ **b** $t = -\frac{6}{7}$ **c** $t = \frac{-1\pm\sqrt{5}}{2}$ **d** impossible

7 Show $\mathbf{a}\bullet\mathbf{b} = \mathbf{b}\bullet\mathbf{c} = \mathbf{a}\bullet\mathbf{c} = 0$ **8** **b** $t = -\frac{5}{6}$

9 $\overrightarrow{AB}\bullet\overrightarrow{AC} = 0$, $\therefore$ $\angle$BAC is a right angle

10 **b** AB = $\sqrt{14}$ units, BC = $\sqrt{14}$ units, ABCD is a rhombus
c 0, the diagonals of a rhombus are perpendicular.

11 **a** 101.3^o or 78.7^o **b** 116.6^o or 63.4^o
c 63.4^o or 116.6^o **d** 71.6^o or 108.4^o

12 **a** 5 **b** -9

13 **a** $k\begin{bmatrix}-2\\5\end{bmatrix}$, $k \neq 0$ **b** $k\begin{bmatrix}-2\\1\end{bmatrix}$, $k \neq 0$ **c** $k\begin{bmatrix}1\\3\end{bmatrix}$, $k \neq 0$
d $k\begin{bmatrix}3\\4\end{bmatrix}$, $k \neq 0$ **e** $k\begin{bmatrix}0\\1\end{bmatrix}$, $k \neq 0$

EXERCISE 15J.2

1 **a** -1 **b** 109.5^o (acute 70.5^o) **c** $\begin{bmatrix}-\frac{1}{3}\\-\frac{1}{3}\\-\frac{1}{3}\end{bmatrix}$ **d** $\frac{1}{\sqrt{3}}$

2 $\angle$ABC $\div 62.5^o$, the exterior angle 117.5^o

3 **a** 54.7^o **b** 60^o **c** 35.3^o

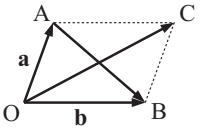
4 **a** 30.3^o **b** 54.2^o **5** **a** M$(\frac{3}{2}, \frac{5}{2}, \frac{3}{2})$ **b** 51.5^o

6 **a** $t = 0$ or -3 **b** $r = -2$, $s = 5$, $t = -4$

7 **a** 74.5^o **b** 72.45^o

8 $\mathbf{a} = \begin{bmatrix}1\\0\\0\end{bmatrix}$, $\mathbf{b} = \begin{bmatrix}0\\1\\0\end{bmatrix}$, $\mathbf{c} = \begin{bmatrix}0\\0\\1\end{bmatrix}$ will do
$\mathbf{a}\bullet\mathbf{b} = \mathbf{a}\bullet\mathbf{c}$, but $\mathbf{b} \neq \mathbf{c}$

10 **a** **Hint:** Square both sides.
b Consider the parallelogram. Find $\overrightarrow{AB}$ and $\overrightarrow{OC}$, etc.



11 -7

12 $\mathbf{a}\bullet\mathbf{b}$ is a scalar and so $\mathbf{a}\bullet\mathbf{b}\bullet\mathbf{c}$ is a scalar 'dotted' with a vector which is meaningless.

EXERCISE 15K.1

1 **a** [2, 5, 11] **b** [2, 4, 1] **c** $-\mathbf{i} - \mathbf{j} - \mathbf{k}$ **d** $\mathbf{i} - 6\mathbf{j} + 2\mathbf{k}$

2 **a** $\mathbf{a}\times\mathbf{b} = [-11, -2, 5]$,
$\mathbf{a}\bullet(\mathbf{a}\times\mathbf{b}) = 0 = \mathbf{b}\bullet(\mathbf{a}\times\mathbf{b})$
$\mathbf{a}\times\mathbf{b}$ is a vector perpendicular to both $\mathbf{a}$ and $\mathbf{b}$

3 **a** $\mathbf{i}\times\mathbf{i} = \mathbf{0}$ $\mathbf{j}\times\mathbf{j} = \mathbf{0}$ $\mathbf{k}\times\mathbf{k} = \mathbf{0}$
b $\mathbf{i}\times\mathbf{j} = \mathbf{k}$ $\mathbf{j}\times\mathbf{i} = -\mathbf{k}$ $\mathbf{j}\times\mathbf{k} = \mathbf{i}$ $\mathbf{k}\times\mathbf{j} = -\mathbf{i}$
$\mathbf{i}\times\mathbf{k} = -\mathbf{j}$ $\mathbf{k}\times\mathbf{i} = \mathbf{j}$
$\mathbf{a}\times\mathbf{a} = \mathbf{0}$ $\mathbf{a}\times\mathbf{b} = -\mathbf{b}\times\mathbf{a}$

5 **a** $\begin{bmatrix}1\\4\\2\end{bmatrix}$ **b** 17 **c** 17

7 **a** $\begin{bmatrix}2\\-1\\-1\end{bmatrix}$ **b** $\begin{bmatrix}0\\5\\0\end{bmatrix}$ **c** $\begin{bmatrix}2\\4\\-1\end{bmatrix}$ **d** $\begin{bmatrix}2\\4\\-1\end{bmatrix}$

8 $\mathbf{a}\times(\mathbf{b}+\mathbf{c}) = (\mathbf{a}\times\mathbf{b}) + (\mathbf{a}\times\mathbf{c})$

11 **a** $\mathbf{a}\times\mathbf{b}$ **b** 0 **c** 0

12 **a** $k\begin{bmatrix}-4\\1\\3\end{bmatrix}$ **b** $k\begin{bmatrix}6\\22\\-15\end{bmatrix}$ **c** $(-\mathbf{i} + \mathbf{j} - 2\mathbf{k})n$
d $(5\mathbf{i} + \mathbf{j} + 4\mathbf{k})n$ $n, k \in \mathcal{R}$, $n, k \neq 0$

13 $k\begin{bmatrix}4\\-5\\-7\end{bmatrix}$, $k \neq 0$, $\frac{\sqrt{10}}{6}\begin{bmatrix}4\\-5\\-7\end{bmatrix}$ *or* $-\frac{\sqrt{10}}{6}\begin{bmatrix}4\\-5\\-7\end{bmatrix}$

14 **a** $\begin{bmatrix}2\\5\\-1\end{bmatrix}$ **b** $\begin{bmatrix}2\\0\\1\end{bmatrix}$

EXERCISE 15K.2

1 **a** $\mathbf{i}\times\mathbf{k} = -\mathbf{j}$, $\mathbf{k}\times\mathbf{i} = \mathbf{j}$

2 **a** $\mathbf{a}\bullet\mathbf{b} = -1$ $\mathbf{a}\times\mathbf{b} = [1, 5, 1]$
b $\cos\theta = -\frac{1}{\sqrt{28}}$ **c** $\sin\theta = \frac{\sqrt{27}}{\sqrt{28}}$ **d** $\sin\theta = \frac{\sqrt{27}}{\sqrt{28}}$

4 **a** $\overrightarrow{OA} = [2, 3, -1]$ $\overrightarrow{OB} = [-1, 1, 2]$
b $\overrightarrow{OA}\times\overrightarrow{OB} = [7, -3, 5]$ $|\overrightarrow{OA}\times\overrightarrow{OB}| = \sqrt{83}$
c Area ΔOAB $= \frac{1}{2}|\overrightarrow{OA}||\overrightarrow{OB}|\sin\theta$
$= \frac{1}{2}|\overrightarrow{OA}\times\overrightarrow{OB}| = \frac{\sqrt{83}}{2}$ units^2

5 **a** $\overrightarrow{OC}$ is parallel to $\overrightarrow{AB}$ **b** $\mathbf{a}\times\mathbf{b} = \mathbf{b}\times\mathbf{c}$

EXERCISE 15K.3

1 **a** $\frac{\sqrt{101}}{2}$ units^2 **b** $\frac{\sqrt{133}}{2}$ units^2 **c** $\frac{\sqrt{69}}{2}$ units^2

2 $8\sqrt{2}$ units^2 **3** **a** D$(-4, 1, 3)$ **b** $\sqrt{307}$ units^2

4 **a** 4 units^3 **b** $(\sqrt{42} + 2\sqrt{3} + 3\sqrt{2} + 6)$ units^2

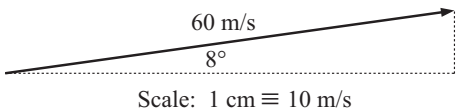
5 **a** (3, 1, 0), (1, 3, 3), (4, 2, 3), (4, 3, 3) **b** $\div 79.01^o$
c 9 units^3 **6** $k = 2 \pm 2\sqrt{33}$

7 $S = \frac{1}{2}\{|\mathbf{a}\times\mathbf{b}| + |\mathbf{a}\times\mathbf{c}| + |\mathbf{b}\times\mathbf{c}| + |(\mathbf{b}-\mathbf{a})\times(\mathbf{c}-\mathbf{a})|\}$

9 **a** Yes **b** No **10** $k = \frac{23}{10}$

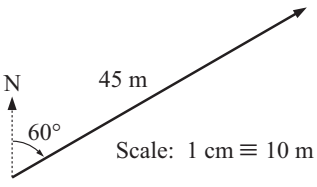
REVIEW SET 15A

1 **a**



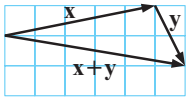
Scale: 1 cm ≡ 10 m/s

b

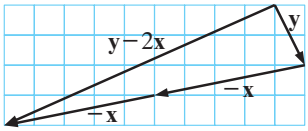


Scale: 1 cm ≡ 10 m

2 **a**



b



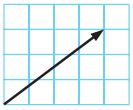
3 **a** $\overrightarrow{PQ}$ **b** $\overrightarrow{PR}$ **4** 4.845 km, 208^o **5** **a** $\overrightarrow{AC}$ **b** $\overrightarrow{AD}$

6 **a** $AB = \frac{1}{2}CD$, $AB \parallel CD$ **b** C is midpoint AB

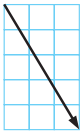
7 **a** $\mathbf{p} + \mathbf{r} = \mathbf{q}$ **b** $\mathbf{l} + \mathbf{m} = \mathbf{k} - \mathbf{j} + \mathbf{n}$

8 **a** $\mathbf{r} + \mathbf{q}$ **b** $-\mathbf{p} + \mathbf{r} + \mathbf{q}$ **c** $\mathbf{r} + \frac{1}{2}\mathbf{q}$ **d** $-\frac{1}{2}\mathbf{p} + \frac{1}{2}\mathbf{r}$

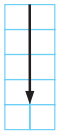
9 **a** $\begin{bmatrix} 4 \\ 3 \end{bmatrix}$



b $\begin{bmatrix} 3 \\ -5 \end{bmatrix}$



c $\begin{bmatrix} 0 \\ -4 \end{bmatrix}$



10 **a** $\begin{bmatrix} -4 \\ -2 \end{bmatrix}$ **b** $\begin{bmatrix} -1 \\ -13 \end{bmatrix}$ **c** $\begin{bmatrix} -4 \\ 8 \end{bmatrix}$ **11** $\begin{bmatrix} 1 \\ 4 \end{bmatrix}$

12 **a** $\sqrt{17}$ units **b** $\sqrt{13}$ units **c** $\sqrt{10}$ units **d** $\sqrt{109}$ units

13 **a** $\mathbf{p} + \mathbf{q}$ **b** $\frac{3}{2}\mathbf{p} + \frac{1}{2}\mathbf{q}$

14 **a** $\mathbf{x} = \begin{bmatrix} -1 \\ \frac{1}{3} \end{bmatrix}$ **b** $\mathbf{x} = \begin{bmatrix} 1 \\ -10 \end{bmatrix}$ **16** $r = 4,\ s = 7$

17 **a** $\mathbf{q} + \mathbf{r}$ **b** $\mathbf{r} + \mathbf{q}$, $DB = AC$, $DB \parallel AC$

REVIEW SET 15B

1 **a** $\overrightarrow{PQ} = \begin{bmatrix} -3 \\ 12 \\ 3 \end{bmatrix}$ **b** $\sqrt{162}$ units **c** $\sqrt{61}$ units

2 **a** $\begin{bmatrix} 3 \\ -3 \\ 11 \end{bmatrix}$ **b** $\begin{bmatrix} 7 \\ -3 \\ -26 \end{bmatrix}$ **c** $\sqrt{74}$ units **3** $\begin{bmatrix} 8 \\ -8 \\ 7 \end{bmatrix}$

4 $m = 5,\ n = -\frac{1}{2}$ **5** 2 : 3 **6** $t = 2 \pm \sqrt{2}$ **7** 80.3^o

8 40.7^o **9** **a** $\begin{bmatrix} -6 \\ 1 \\ 3 \end{bmatrix}$ **b** $\sqrt{46}$ units **c** $(-1, 3\frac{1}{2}, \frac{1}{2})$

10 **a** -1 **b** $\begin{bmatrix} 4 \\ -1 \\ 7 \end{bmatrix}$ **c** 60^o

11 $\measuredangle K \div 123.7^o$, $\measuredangle L \div 11.3^o$, $\measuredangle M \div 45.0^o$

12 63.95^o **13** $c = \frac{50}{3}$

14 **a** $\mathbf{a} \bullet \mathbf{b}$ is a scalar, so $\mathbf{a} \bullet \mathbf{b} \bullet \mathbf{c}$ is a scalar dotted with a vector, which is meaningless.

b $\mathbf{b} \times \mathbf{c}$ must be done first otherwise we have a scalar crossed with a vector which is meaningless.

15 **a** $k = \pm\frac{7}{\sqrt{33}}$ **b** $k = \pm\frac{1}{\sqrt{2}}$

REVIEW SET 15C

1 **a** -13 **b** -36 **3** $t = \frac{2}{3}$ or -3 **4** $k = 6$

5 $k\begin{bmatrix} 5 \\ 4 \end{bmatrix}$, $k \neq 0$ **6** $\measuredangle K = 64.44^o$, $\measuredangle L = 56.89^o$, $\measuredangle M = 58.67^o$

7 72.35^o or 107.65^o

8 **a** **i** **(1)** $\mathbf{p} + \mathbf{q}$ **(2)** $\frac{1}{2}\mathbf{p} + \frac{1}{2}\mathbf{q}$

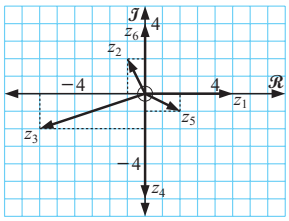
b **i** $\overrightarrow{AC} = -\mathbf{p} + \mathbf{r}$, $\overrightarrow{BC} = -\mathbf{q} + \mathbf{r}$

9 **a** $\begin{bmatrix} 7 \\ -12 \\ -7 \end{bmatrix}$ **b** $\begin{bmatrix} 1 \\ -\frac{5}{3} \\ -\frac{2}{3} \end{bmatrix}$ **c** $\begin{bmatrix} \frac{5}{14} \\ -\frac{5}{7} \\ -\frac{15}{14} \end{bmatrix}$

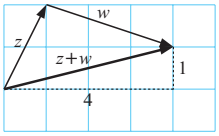
10 **a** ± 7 **b** $\frac{\sqrt{14}}{2}$ units^2 **c** $\frac{7}{6}$ units^3

EXERCISE 16A.1

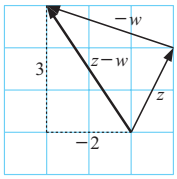
1



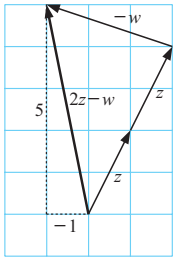
2 **a** $4 + i$



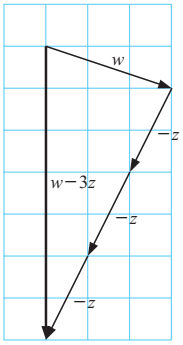
b $-2 + 3i$



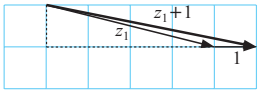
c $-1 + 5i$



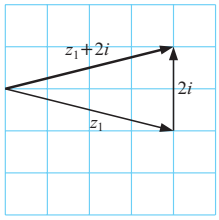
d $-7i$



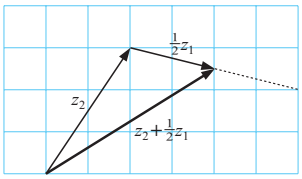
3 **a** $5 - i$



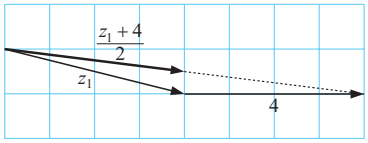
b $4 + i$



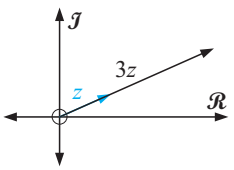
c $4 + \frac{5}{2}i$



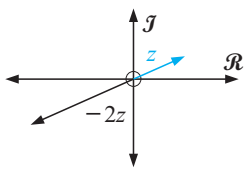
d $4 - \frac{1}{2}i$



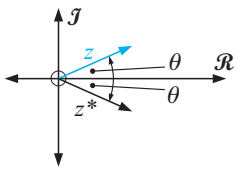
4 **a**



b



c



d